# AP Calculus AB

## Unit 4 – More Differentiation

**AP Calculus AB – Worksheet 32** **Higher Order Derivatives**

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|---|---|
| 1 | Find the first four derivatives of $y=\frac{x^4}{4}+\frac{5}{6}x^3-x^2+9x-1$ . |
| 2 | Find the first and second derivatives of $y=-7x^5-9$. |
| 3 | Find the first and second derivatives of $f(x)=9x^{-3}-\frac{7}{x}$. |
| 4 | Find $\frac{d^2y}{dx^2}$ for $y=\left(3+\frac{5}{x}\right)^3$. |
| 5 | Find $y''$ for $y=x(3x+2)^4$. |
| 6 | Find $f''(x)$ for $f(x)=\frac{x^2-2x}{x+1}$ . |
| 7 | Find an equation of the line perpendicular to the tangent to the curve $f(x)=x^3-9x+3$ at the point $(3,3)$. |
| 8 | Find the points on the curve $f(x)=4x^3+6x^2-72x+18$ where the tangent line is parallel to the *x*-axis. |
| 9 | Evaluate $\frac{d}{dx}\left(f\left(g\left(3x^9\right)\right)\right)$. |

**AP Calculus AB - Worksheet 33** **Linear Approximations, Calculator Derivatives, Higher Order Derivatives**

| | |
|---|---|
| 1. | If $f(x)=x^3+3x$, approximate $f(2.01)$ using linearization centered at $x=2$. |
| 2. | Approximate $\sqrt{24.9}+(24.9)^2$ using linearization. |
| 3. | Find an approximate value for $f(-3.9)$ on $f(x)=\sqrt{x^2+9}$ using linearization. |
| 4. | Approximate using tangent line approximation: $\sqrt[4]{17}$ |
| 5. | Approximate using a tangent line approximation $(8.4)^{4/3}$. |
| 6. | Find $\frac{d^2y}{dx^2}$ for $y=\frac{x+1}{x-2}$. Simplify. |
| 7 | Find $f''(x)$ for $f(x)=x(x+2)^3$. Simplify. (Hint: factor after taking the first derivative to make the second derivative a bit easier) |
| 8. | Evaluate $f'(1)$ for $f(x)=x^{\frac{1}{3}}-x^2+4x$ using a calculator. |
| 9. | Evaluate $f'\left(\frac{\pi}{4}\right)$ for $f(x)=\sin^2 x$ using a calculator. |
| 10 | Evaluate $f'(2)$ for $f(x)=\frac{e^x}{x+2}$ ss using a calculator. |
| 11 | Find all values of x for which the function below is differentiable. $$g(x)=\begin{cases}(x+3)^2, & x\le -2\\ 2x+5, & -2<x<5\\ (6-x)^2, & x\ge 5\end{cases}$$ |

**Answers:**

| | | |
|---|---|---|
| 1. 14.15 | 2. 624.99 | 3. 4.92 |
| 4. 2.031 | 5. $\frac{256}{15}$ | 6. $\frac{d^2y}{dx^2}=\frac{6}{(x-2)^3}$ |
| 7. $f''(x)=12(x+2)(x+1)$ | 8. 2.333 | 9. 1 |

**AP Calculus AB - Worksheet 34** **Derivatives of sine and cosine functions.**

**Know the following theorems:**

$$\frac{d}{dx}\sin\square = \cos\square \cdot \frac{d\square}{dx} \quad \text{and} \quad \frac{d}{dx}\cos\square = -\sin\square \cdot \frac{d\square}{dx}$$

**Find the derivative of each function. Simplify, if necessary.**

| | |
|---|---|
| 1 | $y = \sin\left(3x^2 - 4x\right)$ |
| 2 | $y = -5\cos\left(x^3\right)$ |
| 3 | $f(x) = \cos^2 x$ |
| 4 | $y = \sin x \cos x$ |
| 5 | $y = 2x\cos x$ |
| 6 | $f(x) = \frac{1}{x} + 5\sin x$ |
| 7 | $y = \frac{x}{\cos x}$ |
| 8 | $f(x) = \frac{\cos x}{1+\sin x}$ |
| 9 | $y = \left(1+\cos 2x\right)^3$ |
| 10 | $y = \sin^2\left(3\pi t - 2\right)$ |

| | |
|---|---|
| 11 | Find the equation for the line that is tangent to $f(x) = \sin(x) + 3$ at $x = \frac{\pi}{6}$. |
| 12 | Find the equation of the normal line to $f(x) = \sin x + \cos x$ at $x = \pi$. |
| 13 | Determine all values of $x$ in the interval $(0, 2\pi)$ for which $f(x) = \cos x$ has horizontal tangents. |

**Answers**

| | | |
|---|---|---|
| 1) $y' = (6x-4)\cos\left(3x^2 - 4x\right)$ | 2) $\frac{dy}{dx} = 15x^2\sin\left(x^3\right)$ | 3) $f'(x) = -2\cos x \sin x$ |
| 4) $y' = -\sin^2 x + \cos^2 x$ | 5) $y' = 2\left(\cos x - x\sin x\right)$ | 6) $f'(x) = -\frac{1}{x^2} + 5\cos x$ |
| 7) $y' = \frac{\cos x + x\sin x}{\cos^2 x}$ | 8) $f'(x) = -\frac{1}{1+\sin x}$ | 9) $\frac{dy}{dx} = 3(1+\cos 2x)^2 \cdot (-2\sin 2x)$ |
| 10) $\frac{dy}{dt} = 2\sin(3\pi t - 2)\cos(3\pi t - 2)(3\pi)$ | 11) $T: y - \frac{7}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$ | 12) $y + 1 = x - \pi$ |
| 13) $x = \pi$ | | |

**AP Calculus AB - Worksheet 35** **Derivatives of Trigonometric Functions**

**Know the following Theorems**

| | | | |
|---|---|---|---|
| $\frac{d\tan\square}{dx}=\sec^2\square\cdot\frac{d\square}{dx}$ | $\frac{d\cot\square}{dx}=-\csc^2\square\cdot\frac{d\square}{dx}$ | $\frac{d\sec\square}{dx}=\sec\square\tan\square\cdot\frac{d\square}{dx}$ | $\frac{d\csc\square}{dx}=-\csc\square\cot\square\cdot\frac{d\square}{dx}$ |

**(#1-4 Optional)**
**Use the **quotient rule** to **<u>prove</u>** the derivative of: [Hint: change into sin x and cos x and then take the derivative]**

1. $\tan x$ 2. $\cot x$ 3. $\sec x$ 4. $\csc x$

| | |
|---|---|
| 5. Find $f'(x)$ for $f(x)=11x+3\cos x$ | 6. Find $\frac{dy}{dx}$ for $y=\frac{5\cos x}{1-\sin x}$ |
| 7. Find a linearization of $f(x)$ at a suitably chosen integer near $a$. Use the linearization to approximate $f(a)$. $f(x)=3x^2+9x-3$; $a=-0.9$ | |
| 8. Use a linear approximation for estimate $\sqrt{147}$. | |
| 9. Estimate $f(2.9)$ given that $f(3)=5$ and $f'(3)=6$. | |
| 10. Find $y''$ for $y=3x\sin x$. | 11. Find $f'(x)$ for $f(x)=6\sin x\cos x$ |
| 12. Find the derivative of $f(x)=4\sec x+5\cot x$. | 13. Find $y''$ for $y=8\cot x$. |
| 14. Find the derivative of $f(x)=\csc x\sec x$. | 15. Find $f'(x)$ for $f(x)=\tan 3x-\cot 3x$ |
| 16. Find $f'(x)$ for $f(x)=\frac{\tan x}{\cos x-4}$. | 17. Find $f'(x)$ for $f(x)=\sec x(\tan x)$ |

**AP Calculus AB - Worksheet 36** **Derivatives of Exponential and Logarithmic Functions**

**Know the following theorems:**

| | | | |
|---|---|---|---|
| $\frac{d}{dx}\left(e^{\square}\right)=e^{\square}\cdot d\square$ | $\frac{d}{dx}\left(b^{\square}\right)=b^{\square}\cdot d\square\cdot\ln b$ | $\frac{d}{dx}\left(\ln\square\right)=\frac{d\square}{\square}$ | $\frac{d}{dx}\left(\log_b\square\right)=\frac{d\square}{\square\cdot\ln b}$ |

**Find** $\frac{dy}{dx}$ **.**

**1.** $y=2e^x$

**2.** $y=e^{2x}$

**3.** $y=\log_3(\sin x)$

**4.** $y=e^{-5x}$

**5.** $y=e^{2x/3}$

**6.** $y=\ln\left(x^3\right)$

**7.** $y=xe^x$

**8.** $y=x^2e^x-xe^x$

**9.** $y=4^{\sqrt{x}}$

**10.** $y=5^{\left(x^2\right)}$

**11.** $y=\ln(\cos x)$

**12.** $y=(\ln x)^2$

**13.** $y=(\ln x+\sin x)^4$

**14.** $y=\ln\left(\frac{10}{x}\right)$

**15.** $y=\log_4\left(x^2-3x+\cos x\right)$

**16.** $y=x\ln x-x$

**17.** Find $y''$ for $y=x^2e^x$.

**18.** Find an equation of the tangent line to the graph of the given function at the indicated $x$-value.

a) $f(x)=3x^2-\ln x,\quad x=1$

b) $f(x)=\ln(1+\sin x),\quad x=\frac{\pi}{4}$

**AP Calculus AB – Worksheet 37** **L'Hopital's Rule and Quiz Review**

| | |
|---|---|
| L'Hopital's Rule: | Suppose that f(a)=g(a)=0, that f & g are differentiable on an open interval containing a, and that $g'(x) \neq 0$ on ; then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ |
| Examples: | $\lim_{x\to 2}\frac{x^2-4}{x-2} = \frac{0}{0} \Rightarrow$ Use L'Hopital's Rule - Take derivative of numerator and denominator $\Rightarrow$ $\lim_{x\to 2}\frac{2x}{1} = \boxed{4}$ |
| | $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1} = \frac{0}{0} \Rightarrow$ Use L'Hopitals: $\lim_{x\to 1}\frac{1}{\frac{1}{2\sqrt{x}}} = \lim_{x\to 1}\frac{2\sqrt{x}}{1} = \boxed{2}$ |
| | $\lim_{x\to 2}\frac{x^2-4}{x+2} = \frac{0}{4} = \boxed{0} \Rightarrow$ You cannot use L'Hopital's Rule on this problem! Why?? |

Evaluate each Limit. Use L'Hopital's Rule where appropriate.

| | | | |
|---|---|---|---|
| 1 | $\lim_{x\to 2}\frac{x^3-8}{x^2-4}$ | 2 | $\lim_{x\to 0}\frac{\sin 5x}{x}$ |
| 3 | $\lim_{x\to 2}\frac{\sqrt{2+x}-2}{x-2}$ | 4 | $\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}$ |
| 5 | $\lim_{x\to 2}\frac{x^2-4x+4}{x^3-12x+16}$ | 6 | $\lim_{x\to \frac{\pi}{2}}\frac{1-\sin x}{1+\cos 2x}$ |
| 7 | $\lim_{x\to 1}\frac{x^3-1}{4x^3-x-3}$ | 8 | $\lim_{x\to 3}\frac{x-4}{x-2}$ |

Find each derivative.

| | | | |
|---|---|---|---|
| 9 | $f(x) = e^{\sqrt{x}}$ | 10 | $y = e^{4\ln x}$ |
| 11 | $y = \ln\lvert\sin x\rvert$ | 12 | $y = \sqrt{4x^2+4x}$ |
| 13 | $g(x) = \cos^3 5x$ | 14 | $y = xe^{2x}$ |

| | |
|---|---|
| 15 | Use a tangent line approximation to estimate the value of $\sqrt[3]{25}$. |
| 16 | For $f(x) = e^x$, use a tangent line approximation centered at $x = 0$ to estimate $f(0.1)$. |

**Answers:**

| | | | |
|---|---|---|---|
| 1) 3 | 2) 5 | 3) $\frac{1}{4}$ | 4) $\frac{1}{3}$ |
| 5) $\frac{1}{6}$ | 6) $\frac{1}{4}$ | 7) $\frac{3}{11}$ | 8) -1 |

| | | | |
|---|---|---|---|
| 9) $f'(x) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ | 10) $y' = 4x^3$ | 11) $y' = \cot x$ | 12) $y' = \frac{2x+1}{\sqrt{x^2+x}}$ |
| 13) $g'(x) = -15\cos^2 5x \sin 5x$ | 14) $y' = e^{2x}(2x+1)$ | 15) $\frac{79}{27}$ | 16) 1.1 |

| | |
|---|---|
| 1 | Find $\frac{dy}{dx}$ for $x = \csc y$ . |
| 2 | For $x^5 + y^5 = 0$ .<br>a) Find $\frac{dy}{dx}$ .<br>b) Find the slope of the curve at the point $(3,-3)$. |
| 3 | For $y^3 = 27x$:<br>a) Find $\frac{dy}{dx}$ .<br>b) Find the slope of the curve at the point $(1,3)$. |
| 4 | For $\cos(2y) = x$:<br>a) Find $\frac{dy}{dx}$ .<br>b) Find the slope of the curve at the point $\left(0,\frac{\pi}{4}\right)$. |
| 5 | For $12x + 2y^2 = 8$, find $\frac{d^2y}{dx^2}$ in terms of *x* and *y*. |
| 6 | Find the slope of the curve $5y^8 + 7x^7 = 9y + 3x$ at the point $(1,1)$. |
| 7 | Find the equation tangent and normal lines to the curve $x^2 + xy - y^2 = -1$ at the point $(3,5)$. |
| 8 | Find the equations of the tangent lines to the curve $x^2 + y^2 = 5$ at the points when $x = -2$. |

**Answers**

| | | |
|---|---|---|
| 1) $\frac{dy}{dx} = -\sin y \tan y$ | 2) a) $\frac{dy}{dx} = -\frac{x^4}{y^4}$<br>b) $-1$ | 3) a) $\frac{dy}{dx} = \frac{9}{y^2}$<br>b) 1 |
| 4) a) $\frac{dy}{dx} = -\frac{1}{2\sin(2y)}$<br>b) $-\frac{1}{2}$ | 5) $\frac{d^2y}{dx^2} = -\frac{9}{y^3}$ | 6) $-\frac{46}{31}$ |
| 7) T: $y-5 = \frac{11}{7}(x-3)$<br>N: $y-5 = -\frac{7}{11}(x-3)$ | 8) $y-1 = 2(x+2)$<br>$y+1 = -2(x+2)$ | |

**AP Calculus AB – Worksheet 39** **Implicit Differentiation**

Find $\frac{dy}{dx}$.

| | |
|---|---|
| 1 | $x^2y+xy^2=6$ |
| 2 | $y^2=\frac{x-1}{x+1}$ |
| 3 | $x=\tan y$ |
| 4 | $x+\sin y=xy$ |
| 5 | $x^2-xy=5$ |
| 6 | $y=x^{9/4}$ |
| 7 | $y=\sqrt[3]{x}$ |
| 8 | $y=(2x+5)^{-1/2}$ |

| | |
|---|---|
| 9 | For $x^3+y^3=18xy$, show that $\frac{dy}{dx}=\frac{6y-x^2}{y^2-6x}$ |
| 10 | For $x^2+y^2=13$, find the slope of the tangent line at the point $(-2,3)$. |
| 11 | For $x^2+xy-y^2=1$, find the equations of the tangent lines at the point where $x=2$. |
| 12 | Find $\frac{d^2y}{dx^2}$ for $x^2-y^2=9$ in terms of *x* and *y*. |
| 13 | If $y'=x^2+4x-2$, find three possible equations for *y*. |

**Answers:**

| | | |
|---|---|---|
| 1) $\frac{dy}{dx}=-\frac{2xy+y^2}{2xy+x^2}$ | 2) $\frac{dy}{dx}=\frac{1}{y(x+1)^2}$ | 3) $\frac{dy}{dx}=\cos^2 y$ |
| 4) $\frac{dy}{dx}=\frac{1-y}{x-\cos y}$ | 5) $\frac{dy}{dx}=\frac{2x-y}{x}$ | 6) $\frac{dy}{dx}=\frac{9}{4}x^{\frac{5}{4}}$ |
| 7) $\frac{dy}{dx}=\frac{1}{3}x^{-\frac{2}{3}}$ | 8) $\frac{dy}{dx}=-(2x+5)^{-\frac{3}{2}}$ | 10) $\left.\frac{dy}{dx}\right\|_{(x,y)=(-2,3)}=\frac{2}{3}$ |
| 11) Tangents<br>@ $(2,3)$: $y-3=\frac{7}{4}(x-2)$<br>@ $(2,-1)$: $y+1=-\frac{3}{4}(x-2)$ | 12) $\frac{d^2y}{dx^2}=-\frac{9}{y^3}$ | |

**AP Calculus AB - Worksheet 40** **Derivatives of Inverse Trigonometric Functions**

**Know the following Theorems.**

| | | |
|---|---|---|
| $\frac{d}{dx}\left[\arcsin\square\right]=\frac{d\square}{\sqrt{1-\square^2}}$ | $\frac{d}{dx}\left[\arctan\square\right]=\frac{d\square}{1+\square^2}$ | $\frac{d}{dx}\left[\text{arcsec}\,\square\right]=\frac{d\square}{\lvert\square\rvert\sqrt{\square^2-1}}$ |
| $\frac{d}{dx}\left[\arccos\square\right]=\frac{-d\square}{\sqrt{1-\square^2}}$ | $\frac{d}{dx}\left[\text{arccot}\,\square\right]=\frac{-d\square}{1+\square^2}$ | $\frac{d}{dx}\left[\text{arccsc}\,\square\right]=\frac{-d\square}{\lvert\square\rvert\sqrt{\square^2-1}}$ |

**Find the derivative of *y* with respect to the appropriate variable.**

1. $y=\arccos\left(x^2\right)$
2. $y=\arcsin\sqrt{2t}$
3. $y=\arcsin\frac{3}{t^2}$

4*. $y=x\arcsin x+\sqrt{1-x^2}$

5. $y=\text{arcsec}\,5s$
6. $y=\arctan\sqrt{t-1}$

7. Which of the following is $\frac{d}{dx}\arcsin\left(\frac{x}{2}\right)$?

A) $-\frac{2}{\sqrt{4-x^2}}$ B) $-\frac{1}{\sqrt{4-x^2}}$ C) $\frac{2}{4+x^2}$ D) $\frac{2}{\sqrt{4-x^2}}$ E) $\frac{1}{\sqrt{4-x^2}}$

Find the derivative of the function.

8. $y=3-7x^3+3x^7$
9. $y=\frac{2x+1}{2x-1}$
10. $y=\cot\left(\frac{2}{t}\right)$
11. $y=x\sqrt{2x+1}$
12. $r=\tan^2\left(3-\theta^2\right)$
13. $y=\ln\sqrt{x}$
14. $y=xe^{-x}$
15. $y=\ln\left(\sin x\right)$

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**Answers**

**1.** $y'=-\frac{2x}{\sqrt{1-x^4}}$ **2.** $y'=\frac{1}{\sqrt{2t}\sqrt{1-2t}}$ **3.** $y'=-\frac{6}{t\sqrt{t^4-9}}$ **4.** $y'=\sin^{-1}x$

**5.** $y'=\frac{1}{\lvert s\rvert\sqrt{25s^2-1}}$ **6.** $y'=\frac{1}{2t\sqrt{t-1}}$ **7.** E **8.** $y'=-21x^2+21x^6$

**9.** $y'=-\frac{4}{(2x-1)^2}$ **10.** $y'=\frac{2\csc^2\left(\frac{2}{t}\right)}{t^2}$ **11.** $y'=\frac{3x+1}{\sqrt{2x+1}}$ **12.** $y'=-4\theta\tan\left(3-\theta^2\right)\sec^2\left(3-\theta^2\right)$

**13.** $y'=\frac{1}{2x}$ **14.** $y'=-e^{-x}\left(x-1\right)$ **15.** $y'=\cot x$

**Find each derivative.**

| | |
|---|---|
| 1 | $f(x)=e^{x^3}$ |
| 2 | $y=\ln(\csc 3x)$ |
| 3 | $f(x)=\ln\left(x^2+3-e^{5x}\right)$ |
| 4 | $f(x)=\sec^3(2x)$ |
| 5 | $y=\ln(x\sin x)$ |
| 6 | $y=2^{\cos x}$ |
| 7 | $y=x^2e^x$ |
| 8 | $y=\sqrt{x^2+4x+2}$ |

| | |
|---|---|
| 9 | $\lim_{h\to 0}\frac{\cot(x+h)-\cot x}{h}$ |
| 10 | What is the slope of the line tangent to the graph of $y=e^{\sin x}+x$ when $x=\pi$ ? |
| 11 | Approximate $\sqrt[3]{7.6}$ using linearization. |
| 12 | For $f(x)=x^2+2x+8$, approximate $f(2.9)$. |
| 13 | Evaluate $\lim_{x\to 3}\frac{\tan(x-3)}{3e^{x-3}-x}$ |

**Answers:**

| | | | |
|---|---|---|---|
| 1) $f'(x)=3x^2e^{x^3}$ | 2) $y'=-3\cot 3x$ | 3) $f'(x)=\frac{2x-5e^{5x}}{x^2+3-e^{5x}}$ | 4) $f'(x)=6\sec^3 2x\tan 2x$ |
| 5) $y'=\frac{x\cos x+\sin x}{x\sin x}$ | 6) $y'=-2^{\cos x}(\ln 2)\sin x$ | 7) $y'=xe^x(x+2)$ | 8) $y'=\frac{x+2}{\sqrt{x^2+4x+2}}$ |
| 9) $-\csc^2 x$ | 10) 0 | 11) $\frac{59}{30}$ | 12) $f(2.9)\approx 22.2$ |
| 13) $\frac{1}{2}$ | | | |

**1.** What is the slope of the line tangent to the graph of $y=\dfrac{e^{-x}}{x+1}$ at $x=1$?

**2.** Find the equation of the tangent line to $f(x)=(2x-1)^4(x+3)$ when $x=0$.

**3.** If $f(x)=\cos^3(4x)$, then $f'(x)=$

**4.** $\lim\limits_{h\to 0}\dfrac{e^{(x+h)}-e^x}{h}=$

**5.** For the function *f*, $f'(x)=2x+1$ and $f(1)=4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of *f* at $x=1$?

**6.** If $x^2y-3x=y^3-3$, then at the point $(-1,2)$, $\dfrac{dy}{dx}=$

**7.** $y=\arcsin(5x)$, then $\dfrac{dy}{dx}=$

**8.** If $y=x\sin x$, then $\dfrac{dy}{dx}=$

**9.** If $y=\left(x^3-\cos x\right)^5$, then $y'=$

**10.** If $f(x)=\sqrt{x^2-4}$ and $g(x)=3x-2$, then the derivative of $f(g(x))$ at $x=3$ is

**11.** The function *f* is defined by $f(x)=\dfrac{x}{x+2}$. Find points $(x,y)$ on the graph of *f* that the line tangent to *f* at $(x,y)$ has slope $\dfrac{1}{2}$?

**12.** If $x^2-2xy+3y^2=8$, then show that $\dfrac{dy}{dx}=\dfrac{y-x}{3y-x}$.

**13.** An equation of the line normal to the graph of $y=\sqrt{3x^2+2x}$ at $(2,4)$ is